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SOME FINITE SIMPLE GROUPS OF LIE TYPE $C_n(q)$ ARE UNIQUELY DETERMINED BY THEIR ELEMENT ORDERS AND THEIR ORDER

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Abstract

For a finite group G, denote by $\omega(G)$ the set of the element orders in G.

We show the validity of a conjecture of Shi for the finite simple groups of Lie type $C_n(q)$, where q is an even prime power and $n \geq 16$.

Theorem. Let G be a finite group and M be one of the finite simple groups of Lie type $C_n(q)$, where q is an even prime power and $n \geq 16$. If $|G| = |M|$ and $\omega(G) = \omega(M)$, then $G \simeq M$.

1. Introduction

For a finite group G, denote by $\omega(G)$ the set of element orders in G. A finite group G is said to be *recognizable* by the set of its element orders

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(briefly, recognizable), if every finite group H with $\omega(G) = \omega(H)$ is isomorphic to G.

A finite non-abelian simple group S is said to be *quasirecognizable*, if every finite group H with $\omega(S) = \omega(H)$ has a unique non-abelian composition factor and this factor is isomorphic to S.

We can apply the Gruenberg-Kegel theorem (see Lemma 2.1), when establishing the certain quasirecognizable property of these groups with disconnected prime graphs. In [1, 2], it was shown that the finite simple non-abelian groups P with the number of connected components $s(P) \geq 3$, are quasirecognizable except when P is isomorphic to group A_6. In [10], it was proved that the simple group P, with the number of connected components $s(P) = 2$, is quasirecognizable when P is one of simple groups ${}^2D_{2^m}(2^k)$, ${}^2D_{2^m+1}(2)(m > 1)$, and $C_{2^m}(2^k)(m > 2)$.

We can not apply Lemma 2.1 when establishing the certain quasirecognizable property of the simple groups $C_n(q)$, where q is an even prime power and $n \geq 16$, which have connected prime graphs. But, Lemma 2.2 allows us to start proving quasirecognizability under much weaker conditions on the group under consideration. In this way, the simple linear groups $L_n(2^m)$ with $n = 2^l \geq 16$ (see [5, 13]) were proved to be recognizable. We obtain a list of the finite simple groups S with $t(S) \geq 12$ and $t(2, S) \geq 3$ in this paper.

Shi in [8] put forward the following conjecture:

Conjecture. Let G be a finite group and M be a finite simple group. Then $G \simeq M$, if and only if $|G| = |M|$ and $\omega(G) = \omega(M)$.

This conjecture is correct for Z_p, where p is a prime number, A_n, where $n \geq 5$, sporadic simple groups, simple groups of Lie type except $B_n(q)$, $C_n(q)$, and $D_n(q)$[15], where q is a prime power, and simple groups with order less than 10^8. We shall show the validity of this conjecture for the finite simple groups of Lie type $C_n(q)$, where q is an even prime power and $n \geq 16$.

Theorem. *Let G be a finite group and M be one of the finite simple groups of Lie type $C_n(q)$, where q is an even prime power and $n \geq 16$. If $|G| = |M|$ and $\omega(G) = \omega(M)$, then $G \simeq M$.*

All groups considered in this paper are finite groups and simple groups are non-abelian. All the unexplained notations in this paper are standard and can be found in [4].

2. Preliminary Results

Let G be a group and $\omega(G)$ be the set of orders of elements in G. This set is closed and partially ordered by divisibility, and hence is uniquely determined by the set $\mu(G)$ of its elements, which are maximal under divisibility relation. If p is a prime, then the p-period of G is the maximal power of p that belongs to $\omega(G)$.

Let $\pi(G)$ be the set of all prime divisors of order of G. The set $\omega(G)$ of the group G defines a prime graph $GK(G)$, whose vertex set is $\pi(G)$ and two distinct primes $p, q \in \pi(G)$ are adjacent, if $pq \in \omega(G)$. Denote by $s(G)$ the number of connected components in $GK(G)$ and by $\pi_i = \pi_i(G)$, $i = 1, 2, \cdots, s(G)$, the i-th connected component. If $|G|$ is even, then $\pi_1(G)$ will be the connected component of G containing 2. Guided by given graph conception, we say that prime divisors p and r of the order of G are adjacent, if vertices p and r are joined by edge in $GK(G)$. Otherwise, primes p and r are said to be *nonadjacent*. Denote by $t(G)$ the maximal number of primes in $\pi(G)$ pairwise nonadjacent in $GK(G)$. In other words, if $\rho(G)$ is some independent set with the maximal number of vertices in $GK(G)$ (the subset of vertices of a graph is called an *independent set*, if its vertices are pairwise nonadjacent), then $t(G) = |\rho(G)|$. In graph theory, this number is called the *independent number of a graph*. By analogy, we denote by $t(2, G)$ the maximal number of vertices in the independent sets of $GK(G)$ containing 2. If $\rho(2, G)$ is some independent set with the maximal number of vertices in $GK(G)$ containing 2, then $t(2, G) = |\rho(2, G)|$. We call this number the 2-*independent number*.

Gruenberg and Kegel gave the following description for finite groups with disconnected prime graph (see Theorem A in [14]):

Lemma 2.1 (Gruenberg and Kegel)**.** *If G is a finite group with disconnected prime graph*, *then one of the following statements holds*:

(a) $s(G) = 2$, G *is Frobenius group*;

(b) $s(G) = 2, G = ABC$ *is soluble*, *where* A, AB *are normal subgroups in* G, B *is a normal subgroup in* BC, *and* AB, BC *are Frobenius groups*;

(c) $G = P$ *is non-abelian simple*;

(d) *G is an extension of a* $\pi_1(G)$-*group by a simple group P*;

(e) *G is an extension of a group of type* (c) *or* (d) *by a* $\pi_1(G)$-*group*.

In cases (d) *and* (e), $GK(P)$ *is disconnected*, $s(P) \geq s(G)$, *and for each* $i \geq 2$, *there exists* $j \geq 2$ *such that* $\pi_i(G) = \pi_j(P)$.

In this article, we need the following statement, which can be applied to a wide class of finite groups including the groups with connected Gruenberg-Kegel graph (see [12]).

Lemma 2.2. *Let G be a finite group satisfying two conditions*:

(a) *there exist three primes in* $\pi(G)$, *which are pairwise nonadjacent in* $GK(G)$, *that is*, $t(G) \geq 3$;

(b) *there exists an odd prime in* $\pi(G)$, *which is nonadjacent to prime* 2 *in* $GK(G)$, *that is*, $t(2, G) \geq 2$.

Then, there exists a finite non-abelian simple group S *such that*, $S \le \overline{G} = G/K \le Aut(S)$ *for maximal soluble subgroup* K *of* G. *Furthermore*, $t(S) \ge t(G) - 1$ *and one of the following statements holds*:

(1) $S \cong Alt_7$ *or* $L_2(q)$ *for some odd* q *and* $t(S) = t(2, S) = 3$;

(2) *for every prime* p *in* $\pi(G)$ *nonadjacent to* 2 *in* $GK(G)$, *the Sylow* p*-subgroup of* G *is isomorphic to the Sylow* p*-subgroup of* S. *In particular*, $t(2, S) \ge t(2, G)$.

Proof. See the main theorem in [12]. □

We use the following number-theoretic notations. If n is a natural number, then $\pi(n)$ is the set of prime divisors of n. If $p \in \pi(n)$, then $n|_p$ is the maximal p-power that divides n. If q is a natural number, r is an odd prime, and $(q, r) = 1$, then by $e(r, q)$, we denote the smallest natural number m such that $q^m \equiv 1 (\mathrm{mod}\, r)$. Given an odd q, put $e(2, q) = 1$, if $q \equiv 1 (\mathrm{mod}\ 4)$ and put $e(2, q) = 2$, if $q \equiv -1 (\mathrm{mod}\ 4)$.

The following number-theoretic result is important for investigations on the structure of prime graphs of the finite simple groups of Lie type.

Lemma 2.3. *Let* q, s *be natural numbers*, $q \ge 2$, $s \ge 2$. *Then one of the following statements holds*:

(1) *there exists a prime* r, *which divides* $q^s - 1$, *but does not divides* $q^t - 1$ *for all natural numbers* $t < s$;

(2) $s = 6$ *and* $q = 2$;

(3) $s = 2$ *and* $q = 2^m - 1$ *for some natural number* m.

Proof. See [16]. □

The prime r with $e(r, q) = s$ is called a *primitive prime divisor of* $q^s - 1$. If q is fixed, we denote by r_s any primitive prime divisor of $q^s - 1$ (obviously, $q^s - 1$ can have more than one primitive prime divisor).

We obtain the following table of the finite simple groups S with $t(S) \geq 12$ and $t(2, S) \geq 3$.

Lemma 2.4. *Let S be a finite simple non-abelian group with* $t(S) \geq 12$ *and* $t(2, S) \geq 3$. *Then* S, $t(2, S)$, $\rho(2, S)$, *and* $t(S)$ *are as in Table* 1.

Proof. See the table in [5]. $\square$

Lemma 2.5. *Let* G *be a finite group,* $N \triangleleft G$ *such that* G / N *is a Frobenius group with kernel F and cyclic complement C. If* $(|F|, |N|) = 1$ *and* F *is not contained in* $NC_G(N)/N$, *then* $p \cdot |C| \in \omega(G)$ *for some prime divisor of* $|N|$.

Proof. See Lemma 1 in [7]. $\square$

Table 1. Simple non-abelian groups S with $t(S) \geq 12$ and $t(2, S) \geq 3$

S	Additional conditions on S	$t(2, S)$	$\rho(2, S) \setminus \{2\}$	$t(S)$
Alt_n	n, $n-2$ are primes	3	$\{n, n-2\}$	
$n \geq 103$	$n-1$, $n-3$ are primes	3	$\{n-1, n-3\}$	
$A_{n-1}(q)$	$2 < (q-1)\|_2 = n_2$	3	$\{r_{n-1}, r_n\}$	$[\frac{n+1}{2}]$
$n \geq 23$	q even	3	$\{r_{n-1}, r_n\}$	
$^2A_{n-1}(q)$	$2 < (q+1)\|_2 = n_2$	3	$\{r_{2n-2}, r_n\}$	$[\frac{n+1}{2}]$
$n \geq 23$	q even, $n \equiv 0 \pmod 4$	3	$\{r_{2n-2}, r_n\}$	
	q even, $n \equiv 1 \pmod 4$	3	$\{r_{n-1}, r_{2n}\}$	
	q even, $n \equiv 2 \pmod 4$	3	$\{r_{2n-2}, r_{n/2}\}$	
	q even, $n \equiv 3 \pmod 4$	3	$\{r_{(n-1)/2}, r_{2n}\}$	
$C_n(q)$, $n \geq 15$	q even	3	$\{r_n, r_{2n}\}$	$[\frac{3n+5}{4}]$
$D_n(q)$	$q \equiv 5 \pmod 8$, $n \equiv 1 \pmod 2$	3	$\{r_{2n-2}, r_n\}$	$[\frac{3n+1}{4}]$
$n \geq 16$	q even, $n \equiv 0 \pmod 2$	3	$\{r_{2n-2}, r_{n-1}\}$	
	q even, $n \equiv 1 \pmod 2$	3	$\{r_n, r_{2n-2}\}$	
$^2D_n(q)$	$q \equiv 3 \pmod 8$, $n \equiv 1 \pmod 2$	3	$\{r_{2n-2}, r_{2n}\}$	$[\frac{3n+4}{4}]$
$n \geq 15$	q even, $n \equiv 0 \pmod 2$	3	$\{r_{n-1}, r_{2n-2}, r_{2n}\}$	
	q even, $n \equiv 1 \pmod 2$	3	$\{r_{2n-2}, r_{2n}\}$	

Lemma 2.6. *The finite simple group* $A_n(q)$, $n > 3$, $q \geq 2$ *includes a Frobenius subgroup with kernel of order* q^n *and cyclic complement of order* $q^n - 1$.

Proof. See Lemma 2.3 in [10]. □

3. Proof of the Theorem

Let $L = C_n(q)$, where $n \geq 16$ and $q = 2^k \geq 2$. By Lemma 2.4, $t(L) \geq 13$, $t(2, L) \geq 3$, and the 2-period of L is equal to 2^m, where m is the smallest natural number such that $2n \leq 2^m$, i.e., $m = 1 + \lceil \log_2 n \rceil$.

Let G be finite group with $\omega(G) = \omega(L), |G| = |L|$, and K be the maximal normal soluble subgroup of G. By Lemma 2.2, there exists a finite non-abelian simple group S such that $S \leq \overline{G} = G / K \leq Aut(S)$; moreover, $t(S) \geq t(G) - 1$ and either $t(S) = t(2, S) = 3$ or $t(2, S) \geq t(2, G)$. Since $t(G) = t(L) \geq 13$, $t(2, G) = t(2, L) \geq 3$, and the 2-period of L is equal to 2^m, where $m = 1 + \lceil \log_2 n \rceil$, the group S must satisfy $t(S) \geq 12$, $t(2, S) \geq 3$, and the 2-period of $S \leq 2^m$, where $m = 1 + \lceil \log_2 n \rceil$. By Lemma 2.4, S is one of the groups in Table 1. The proof relies on analysis of all possibilities for S from this table case by case.

If not specified, r_n and r_{2n} are some fixed primitive prime divisors of $q^n - 1$ and $q^{2n-2} - 1$, respectively. By the definition of primitive prime, these numbers are pairwise distinct. By Lemma 2.4, the primes r_n and r_{2n} are nonadjacent to 2 in $GK(L)$, and so in $GK(G)$ as well. Consequently, by Lemma 2.2, these primes divide the order of S.

(1) S is not isomorphic to $Alt_{n'}$.

Let $S = Alt_{n'}$. Then $n' \geq 103$ and there are two primes among numbers $n', n' - 1, n' - 2, n' - 3$; these are r_n, r_{2n}. By Lemma 2.4, we have $4r_{2n-2} \overline{\in} \omega(L)$ and $2r_{2n-2} \in \omega(L)$. Suppose that r_{2n-2} divides the order of S. Since S does not contain an element of order $4r_{2n-2}$, it follows that $n' \geq r_{2n-2} \geq n' - 5$. Then, there are three primes among the six consecutive numbers $n', n' - 1, \cdots, n' - 5$; which is impossible since $n' \geq 103$. So r_{2n-2} lies in $\pi(K) \bigcup \pi(Out(S))$. Since $\pi(Out(S)) = \{2\}$, we have $r_{2n-2} \in \pi(K)$.

Denote r_{2n-2} by r. Let $\widetilde{G} = G / O_{r'}(K)$ and $\widetilde{K} = K / O_{r'}(K)$. Then $R = O_r(\widetilde{K}) \neq 1$. Suppose that $\widetilde{K} = R$. The group S acts faithfully on $\widetilde{K}$. Otherwise, by its simplicity S centralizes $\widetilde{K}$ and, therefore, G contains an element of order $4r$. The group Alt_6, and so S as well, includes a Frobenius group F with kernel of order 9 and cyclic complement of order 4. By applying Lemma 2.5 to the preimage of F in $\widetilde{G}$, we find that $4r \in \omega(G)$; a contradiction. Suppose that $\widetilde{K} \neq R$. Then, there is a prime t such that $T = O_t(\widetilde{K} / R)$ is nontrivial. Since $O_{r'}(\widetilde{K}) = 1$, the group T acts faithfully on R. Then T acts faithfully on $\widehat{R} = R / \Phi(R)$, where $\Phi(R)$ is the Frattini subgroup of R, as well. Denote by $\widehat{G}$, the factor group $\widetilde{G} / \Phi(R)$. By Table 8 in [11], at least one of the primes r_n and r_{2n} is nonadjacent to t in $\omega(G)$. Denote this prime by s. Let x be an element of order s in $\widehat{G} / \widehat{R}$. Then $H = T\langle x \rangle$ is a Frobenius subgroup in $\widehat{G} / \widehat{R}$. The preimage of H in $\widehat{G}$ satisfies conditions of Lemma 2.5, hence G contains an element of order $r \cdot s$, which contradicts Table 8 in [11], i.e., the primes r and s are nonadjacent in $\omega(G)$.

(2) S is not isomorphic to $A^{\varepsilon}_{n'-1}(q')$, where q' is odd.

Let $S = A^{\varepsilon}_{n'-1}(q')$, where q' is odd. Then $n'_2 = (q - \varepsilon 1)_2 > 2$ and $t(S) = (n' + 1) / 2$. Since $t(S) \geq t(G) - 1$ and $t(G) = [\frac{3n+5}{4}]$, we have

$(n'+1)/2 \geq [\frac{3n+5}{4}]-1$. Hence, $n' \geq 2[\frac{3n+5}{4}]-3 \geq 23$. Let $m' = \lceil \log_2 n' \rceil$. Since $n' > 2^{m'-1}$, S includes a cyclic subgroup of order $q'^{2^{m'-1}}-1$. By

$$q'^{2^{m'-1}} - 1 = (q'-1)(q'+1)(q'^2+1)\cdots(q'^{2^{m'-2}}+1),$$

we have

$$(q'^{2^{m'-1}} - 1)|_2 = (q'-1)|_2(q'+1)|_2(q'^2+1)|_2\cdots(q'^{2^{m'-2}}+1)|_2 \geq 4\cdot 2^{m'-1} = 2^{m'+1}.$$

Thus $2^{m'+1} \in \omega(S)$. Since the 2-period of $S \leq 2^m$, where $m = 1 + \lceil \log_2 n \rceil$, $1 + \lceil \log_2 n' \rceil \leq 1 + \lceil \log_2 n \rceil$. Therefore, $n' \leq n+1$, which contradicts $n' \geq 2[\frac{3n+5}{4}]-3$.

(3) S is not isomorphic to $D^{\varepsilon}_{n'}(q')$, where q' is odd.

Let $S = D^{\varepsilon}_{n'}(q')$, where q' is odd. Then n' is odd and $t(S) \leq [\frac{3n'+4}{4}]$. Since $t(S) \geq t(G)-1$ and $t(G) = [\frac{3n+5}{4}]$, we have $[\frac{3n'+4}{4}] \geq [\frac{3n+5}{4}]-1$. Hence $n' \geq n-2$. Suppose that $n' \geq n+3$. As S includes the universal covering of $A_{n'-2}(q')$, by repeating the arguments of the preceeding paragraph, we get a contradiction.

Next, suppose that $n' = n-2, n-1, n, n+1, n+2$. As S includes the universal covering of $A_{n'-2}(q')$, by repeating the arguments of the preceeding paragraph, we get a contradiction.

By

$$q'^{2^{m-1}} - 1 = (q'-1)(q'+1)(q'^2+1)\cdots(q'^{2^{m-2}}+1),$$

we have

$$(q'^{2^{m-1}} - 1)|_2 = (q'-1)|_2(q'+1)|_2(q'^2+1)|_2\cdots(q'^{2^{m-2}}+1)|_2 = 4\cdot 2^{m-1} = 2^{m+1}.$$

Thus $2^{m+1} \in \omega(S)$, where $m = \lceil \log_2 n \rceil$, and the 2-period of S is 2^{1+m}.

We claim $|K|_2 = 1$. Suppose that $|K|_2 \geq 2$. Let $\widetilde{G} = G / O_{2'}(K)$ and $\widetilde{K} = K / O_{2'}(K)$. Then $R = O_2(\widetilde{K}) \neq 1$. The group S acts faithfully on R. Otherwise, by its simplicity S centralizes $\widetilde{K}$ and, therefore, G contains an element of order $2r_{2n}$. By Lemma 2.6, S includes a Frobenius group F with kernel of order $q'^{2^{m-1}}$ and cyclic complement of order $q'^{2^{m-1}} - 1$. By applying Lemma 2.5 to the preimage of F in $\widetilde{G}$, we find that $2^{m+2} \in \omega(G)$; a contradiction.

Let $|S|_2 = 2^l$. Since $|OutS|_2 \leq 2^{1+m}$, where $m = \lceil \log_2 n \rceil$, $n \geq 16$, we have $l \geq kn^2 - (1 + \lceil \log_2 n \rceil) \geq 251$, and $|S| \leq 2^{3l}$. On the other hand, S is one of the groups in Table 1. So, Lemma 2 in [9] implies that S is a simple group of Lie type over a field of characteristic 2, which contradicts $S = D^{\varepsilon}_{n'}(q')$, where q' is odd.

(4) Next, the cases of simple groups of Lie type over fields of characteristic 2 will be considered.

Since $n \geq 16$, we have that $r_{2n}, r_{2(n-1)}, r_{2(n-2)}$, and $r_{2(n-3)}$ exist by Lemma 2.3. It implies that $r_{2n} \in \pi(S)$ by Lemma 2.2. Furthermore, r_{2n}, $r_{2(n-1)}$, $r_{2(n-2)}$, $r_{2(n-3)} \in \rho(L) = \rho(G)$ by Table 8 in [11]. Since at most one prime in $\rho(G)$ divides $|K| \cdot |\overline{G} / S|$, by Proposition 3 in [12], we have r_{2n}, $r_{2(n-2)}$, $r_{2(n-3)} \in \pi(S)$, but $r_{2(n-1)} \overline{\in} \pi(S)$ or r_{2n}, $r_{2(n-1)} \in \pi(S)$.

If $r_{2n}, r_{2(n-2)}, r_{2(n-3)} \in \pi(S)$, but $r_{2(n-1)} \overline{\in} \pi(S)$ hold, then we easily get a contradiction by Lemma 2.3 for S being one of $A_{n'}(q'), {}^2A_{n'}(q')$, $C_{n'}(q'), D_{n'}(q')$, and ${}^2D_{n'}(q')$, where q' is even. Let $S = A_{n'-1}(q')$, where $q' = 2^{k'}$. We have $2nk = n'k', 2(n-2)k = (n'-1)k'$, and $2(n-3)k = (n'-2)k'$, by Lemma 2.3, which are impossible. We similarly deal with ${}^2A_{n'}(q')$, $C_{n'}(q'), D_{n'}(q')$, and ${}^2D_{n'}(q')$, where q' is even. So all of them lead to a contradiction by Lemma 2.3.

Next suppose $r_{2n}, r_{2(n-1)} \in \pi(S)$. Let $S = A_{n'-1}(q')$, where $q' = 2^{k'}$. We have $2nk = n'k'$ and $2(n-1)k = (n'-1)k'$ by Lemma 2.3. It implies that $n = n'$ and $k' = 2k$. Since $|G|_2 = 2^{n^2k}$ and $|S|_2 = 2^{n(n-1)k}$, we have $(|K||\overline{G}/S|)_2 = q^n$. Let $S_2 \in Syl_2K$. If $|K|_2 \geq 2$, then $G = KN_G(S_2)$ by Frattini's argument. It follows that $S = G/K \simeq N_G(S_2)/N_G(S_2) \cap K$. So $r_{2n} \in \pi(N_G(S_2))$ by Lemma 2.2. Let $|x| = r_{2n}$ and $x \in N_G(S_2)$. Then $S_2 < x >$ is a Frobenius by Lemma 2.4. It implies that $r_{2n} | (|S_2| - 1)$ and $|S_2| = q^{2n}$, by Lemma 2.3, which is a contradiction. Now, $|K|_2 = 1$ and $|\overline{G}/S| = q^n || Out(S)| = def$, where $d = (n, q^2 - 1)$, $e = 2$, $f = 2nk$. It follows that $2^{nk} | 4nk$, which is impossible for $n \geq 16$. We similarly deal with ${}^2A_{n'}(q')$, $D_{n'}(q')$, and ${}^2D_{n'}(q')$, where q' is even. So all of them lead to a contradiction by Lemma 2.3.

Finally, let $S \simeq C_{n'}(q')$. We have $2nk = 2n'k'$ and $2(n-1)k = 2(n'-1)k'$ by Lemma 2.3. It follows that $n = n'$, $k = k'$, and $S \simeq L$. It implies that $G \simeq L$ by $|G| = |L|$.

This completes the proof of the theorem.

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